Wind Turbine Power Estimation by Physical Model

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Abstract

Renewable energy is the source of the most sought assurance energy needs. We will present a method for calculating wind turbine power based on physical model and compare it with reported values. The physical model is based on the definition of the power produced by the movement of the point of force application pushing on the blade surface. The geometry of the blade is considered to be simplified to the size of a rectangle, the goal of the physical model being to obtain an estimate only. Results for power output power are in the same range as the reported ones, without parameter adjustments. The physical model of the wind turbine would be a transparent way to estimate, and didactic in training students.

Keywords*:* physical model wind turbine, wind power estimate, wet wind blade area, compared with reported results

Introduction

In the context of the new energy era and new technologies, which have seen increasing development, there is increased interest in fossil fuel consumption and pollution default reduction through renewable energy sources. A representative direction in the use of renewable energy sources refers to the conversion of wind energy into electrical energy based on the use of wind turbines, the development of these conversion systems led to the identification of new innovative solutions characterized by high efficiency, low cost, reliability and adaptability to the conditions of implementation.

Romania is located in an area of interference of air masses with thermo baric contrasts high and has overall good wind energy potential. Relief configuration, partitioning the country, determines wind obvious changes from one region to another, causing uneven distribution of its. Wind energy is an inexhaustible source of energy and not polluting, and in some areas is abundant, as in Dobrogea.

Particular attention was directed toward command and control strategies of power turbine (wind power) connected directly to the electrical energy national network. For this purpose, it is necessary to control the position of the blades of the wind turbines so that the change of power output is reduced in conditions of rather large variability of the wind speed at which the turbine operates, from a few to 20 m/s, or more. The main purpose of command and control strategies developed capability consists of wind turbines to produce electricity at a cost as low as possible by reducing the mechanical components and blade pitch adjustment to achieve maximum power produced by the turbine.

1. Literature review

Wind turbine blades leeward behaviour analysis has been the subject of several modelling studies.

Using energy balance of the air in contact with the blades, it is analysing the wind turbine rotation by the pressure differential between the exposed face and the not exposed face in the air. There is a lift force perpendicular to the direction of movement of the air. These observations apply when the blade is in the wind, the lift force adding or subtracting from the wind force, which acts directly by pushing (Balan, 2007).

Another approach is by considering the movement of the blades comprising the rotor diameter in tube current and the application of Bernoulli 's equation results are obtained on engine speed and the turbine power (Cosoiu, 2008).

By definition of the power relation obtained as the product of force and speed, mechanical action of the air on the blade is developed. It analyses the air disturbance by wind turbine rotor, causing a decrease in wind speed within the rotation blade plane (Wikipedia).

Estimation of wind turbine operation and energy obtained has been introduced by a simplified model as module in MATLAB language, and is very useful in the design of wind turbines (Munteanu, 2006). There are studies to compute the wind turbine power by considering the cube of the wind velocity multiplied by its operating area, air density and a power coefficient (Safta, 2013):

$$P_m = C_p(\lambda,\beta)\cdot\frac{\rho\cdot A}{2}\cdot v^3 \tag{1}$$

Less discussed is the modelling of the hydrodynamic limit layer peeling under wind turbine operation to higher wind speeds, which are qualitatively specified in the experimental studies of velocity profile around the blades (Mandis, 2009). This study aims to exploit this phenomenon in order to restrict the range of powers turbine to a good absorption into the national electrical power grid.

2. Wind turbine principle

Turbine blade movement is determined by the flow of air masses on the surface exposed to the wind blade. This flow takes place from the outside, where exposure to the wind blade is the maximum inward. At sufficiently low speeds, the hydrodynamic boundary layer is located on the surface of the entire blade, and the force resultant from thrust of the masses of air on the blade are concentrated about the geometric centre thereof, and according to pressure variations at the surface of the blade (Figure 1). As wind speed increases, the geometry and orientation of the blade determines the hydrodynamic boundary layer get away, leaving a certain portion of the blade outside the direct action of the wind. The detachment of the boundary layer starts from the hub and increases towards the blade top until it reaches maximum speed supported for reasons of safety. By restricting the area of contact between the air and the blade, the thrust of the air on the blade moves toward the periphery of the rotor (Figure 2), which is an objective in accordance with low power variations. At high wind speeds, the blade is oriented so that the wind speed has an angle to blade surface, forming a tangential component and a component normal to the blade surface, as follows:

$$a_\beta = a\cdot\sin\beta \tag{2}$$

Specific speed is a criterion of similarity between the turbines and is calculated by the relationship

$$n_s = n \cdot \frac{Q_v^{1/2}}{H_m^{3/4}} \qquad (3)$$

High values of specific speed turbine signifies the force of rotation approach near the turbine shaft, versus small values of specific speed means motive force position remote from the centre of rotation of the turbine.

In equations, *n* represents the actual speed of the turbine, which is intended as uniform with respect to variations in wind speed in order to achieve mechanical power into electrical power converted on as constant, knowing that short power fluctuations produce network integration costs in national energy, which prevents lowering the price to conventional energy resources.

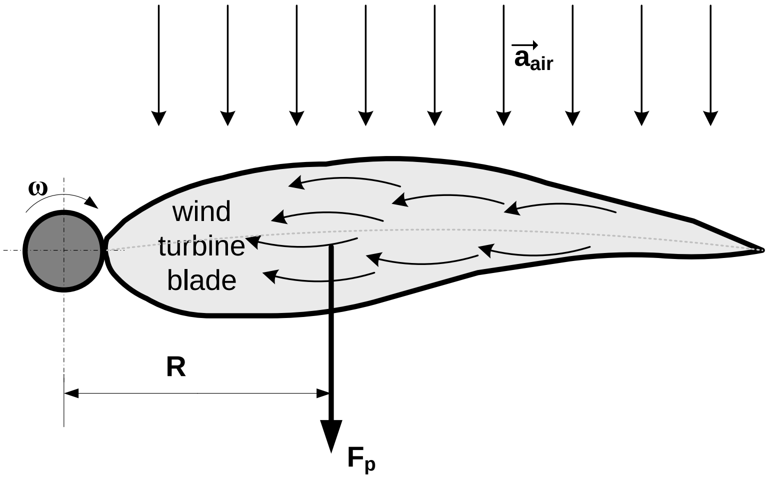

Figure 1. Air flow behaviour around pale under low wind

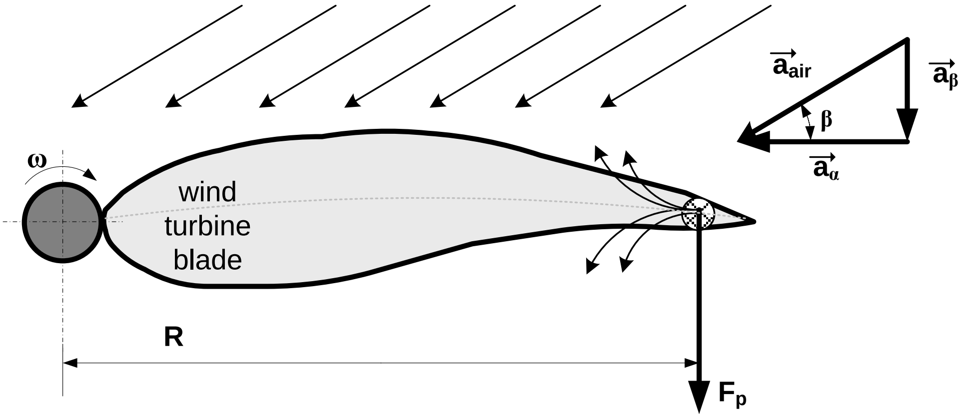

Figure 2 Air flow behaviour around pale under strong wind

Q_v is the volume flow rate of air which strikes the blade and is calculated as the product of the wind speed and blade area found, i.e., "wet", which may be less than the blade surface exposed to the wind.

$$Q_v = a_\beta \cdot S_{b,h} = a \cdot S_{b,h} \cdot \sin\beta \qquad (4)$$

It is considered that air circulation around the blade is not affected by currents from neighbouring blades as the number of blades is chosen so that hydrodynamics around each blade will not interfere with that of a neighbouring blade. As such, where the wind hits the blade, the air velocity is unchanged from that of cruise.

H_m is the gauge height corresponding to power consumption of the air passing over the obstacle represented by hydraulic blade. Assuming that the speed and height above ground of the wind does not change after crossing the blade, the power transferred to the air turbine is the only local pressure drop.

$$H_m = \frac{\Delta p_{local}}{\rho_{air} \cdot g} = \frac{\xi \cdot \frac{\rho_{air} \cdot a^2}{2}}{\rho_{air} \cdot g} = \frac{\xi \cdot a^2}{2 \cdot g} \tag{5}$$

ξ is the local flow resistance coefficient.

It customizes the relation (3) in general for turbines by relations (4) and (5) for particular wind turbine and get:

$$n_s = \left[\frac{2 \cdot g}{\xi}\right]^{3/4} \cdot n \cdot \frac{a^{1/2} \cdot S_{b,h}^{1/2}}{a^{3/2}} \cdot (\sin\beta)^{1/2} = C \cdot n \cdot \frac{S_{b,h}^{1/2}}{a} \cdot (\sin\beta)^{1/2} \tag{6}$$

C group size is constant.

When the wind speed increases, according to the equation (6), the specific speed decreases which means that the drive force of the turbine away from the centre of rotation. This change in the point of application of force can be achieved only by restricting coverage of the natural wind blade surface, from the middle to the top.

3. Numerical example

Consider a wind turbine manufactured by VESTAS V100, which has the following features: blade length, 49 m, maximum blade width, 3.9 m, the minimum allowable wind speed of 3 m/s, speed normal operation, 12 m/s, maximum speed 20 m/s. Air density reported in the datasheet is 1,225 g/cm3. The power produced by the turbine is in the carried chart (Figure 3)

Use conservation of momentum by colliding air perfectly to fine blade, speed vectors having the same direction, perpendicular to the surface of the blade:

$$m_{air} \cdot a_\beta + m_b \cdot v_b = (m_{air} + m_b) \cdot v \tag{7}$$

a_β is the wind speed component perpendicular to the surface of the blade.

Since the mass of air which strikes the blade is endless, blade mass is becoming negligible in relation to the air mass, despite the vice versa density differentia. As such, the balance of momentum (7) follows to

$$v = a_\beta \tag{8}$$

Equation (8) means that the blade speed equals the wind speed component that strikes the blade, by the value before hitting the blade.

The blade is considered shaped and its surface exposed to the wind is assimilated to that of a half-cylinder (Tsakiris, 2013). As a result, the area "wetted" by the air is:

$$S_{b,h} = \left(\frac{\pi}{2} \cdot l_b\right) \cdot L_{b,h} \tag{9}$$

Arm of the force that spins the turbine is:

$$R = L_b - \frac{L_{b,h}}{2} \tag{10}$$

Actual speed of the turbine is:

$$n = \frac{\omega}{2\cdot\pi} = \frac{\frac{a_\beta}{R}}{2\cdot\pi} = \frac{\frac{a_\beta}{\left[L_b - \frac{1}{2}\cdot\left(\frac{2\cdot S_{b,h}}{\pi\cdot l_b}\right)\right]}}{2\cdot\pi} = \frac{a_\beta\cdot l_b}{2\cdot\left(\pi\cdot L_b\cdot l_b - S_{b,h}\right)} \tag{11}$$

The turbine speed is desired to remain constant independently of the wind speed, such as calculating the amount of the reference turbine speed at a wind speed that is determined from the curve in Figure 3, corresponding to the output power constant level. Speed reference value is determined graphically as a_r = 10.4 m/s and reference turbine speed is

$$n = \frac{\frac{a_r}{L_b/2}}{2\cdot\pi} = \frac{\frac{10{,}4}{49/2}}{2\cdot\pi} = 0{,}0673 s^{-1}$$

It approximates the local hydraulic resistance coefficient by:

$$\xi = \sin\beta \tag{12}$$

Indeed, the angle β = 0 is for the wind no longer meets the blade, and the angle β = 90 is for the maximum airflow obstruction.

$$\sin\beta = \frac{a_\beta}{a} \tag{13}$$

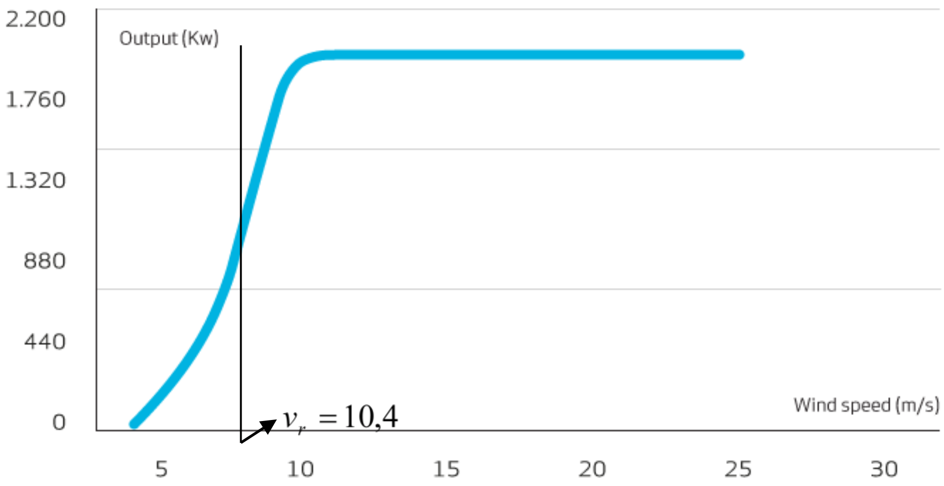

Figure 3. Output reported power of the V100-1.8/2.0 MW wind turbine (VESTAS, 2014)

3.1. Lower wind speeds

For the full range of wind speeds a < a_r, wind turbine blades orient them perpendicular to the wind direction. Since the prevailing wind direction is parallel to the ground, the plane movement blades will be vertical, β = 90 so ξ = sin β = 1.

Power output of the three-bladed wind turbine is calculated as the vector dot product of force and displacement velocity vector of the point of application of force, integrated over the entire surface "wet" to each blade, the blade physical area at the minimum speed of the wind, such as:

$$\begin{aligned}P_3 &= 3\cdot\int_0^{L_b}\vec{a}\cdot d\vec{F}_b = 3\cdot\int_0^{L_b} a\cdot\cos(90-\beta)\cdot dF_b = \\ &= 3\cdot\int_0^{L_b}\frac{\pi}{2}\cdot l_b\cdot\rho_{air}\cdot a^3\cdot(\sin\beta)^3\cdot dL = \\ &= 3\cdot\frac{\pi}{2}\cdot l_b\cdot L_b\cdot\rho_{air}\cdot a^3\cdot(\sin\beta)^3 = \\ &= 3\cdot\frac{\pi}{2}\cdot 3{,}9\cdot 49\cdot 1{,}225\cdot 3^3\cdot 1^3 = \\ &= 29.785\,\mathrm{W} = 29{,}8\,\mathrm{kW}\end{aligned} \qquad (14)$$

Similar, P_4 = 70.6 kW, P_6 = 238.3 kW, P_8 = 564.8 kW, P_{10} = 1103.1 kW.

3.2. Similarity of the wind turbine model

The turbine model is designed to determine parameters that cannot find the actual turbine data. The turbine model has all geometric dimensions scale to the actual turbine, flow 1 m^3/s and gauge height of 1 m and has model fluid movement identical to the actual turbine.

Gauge height of the turbine model is defined by the relation (15):

$$H_{ms} = \frac{\xi\cdot a_{\beta,s}^2}{2\cdot g} \qquad (15)$$

Minimum speed wind turbine model is:

$$a_{\beta,s,\min} = \sqrt{\frac{2\cdot H_{m,s}\cdot g}{\xi_{\max}}} = \sqrt{\frac{2\cdot 1\cdot 9{,}81}{1}} = 4{,}429\,m/s \qquad (16)$$

The value of the minimum speed for the turbine model is independent of actual turbine, regardless of size.

Determine the area of the turbine blade model, S_{max}, which is completely wetted for minimum speed:

$$Q_{vs} = 1m^3/s = a_{\beta,s,\min}\cdot S_{s,\max} \Rightarrow S_{s,\max} = \frac{Q_{v,s}}{a_{\beta,s,\min}} = \frac{1}{4{,}429} = 0{,}2258m^2 \quad (17)$$

Real turbine blade area, S_{max}, is:

$$S_{\max} = \frac{\pi}{2}\cdot l_b\cdot L_b = \frac{\pi}{2}\cdot 3{,}9\cdot 49 = 300{,}18m^2 \qquad (18)$$

Determine the ratio of geometric similarity between the actual turbine and the turbine model:

$$m^2 = \frac{S_{\max}}{S_{s,\max}} = \frac{300{,}18}{0{,}2258} = 1329{,}6 \Rightarrow m = 36{,}464 \qquad (19)$$

Considering a model of the turbine to calculate the turbine power is required for the wind speed values of above the reference, when the plane of rotation of the blade is no more oriented vertically, but it is becoming more and more to the horizontal with increasing wind speed.

3.3. Higher wind speeds

Winds decomposition is made between blade perpendicular component, $a_\beta = a_r = 10.4$ m/s and the component parallel to the blade

$$a_\alpha = \sqrt{a^2 - a_\beta^2} = \sqrt{12^2 - 10{,}4^2} = 6{,}05\, m/s.$$

The actual wind speed decomposition is to start calculus and will be verified.

$$\sin\beta = \frac{a_\beta}{a} = \frac{10{,}4}{12} = 0{,}867 \qquad (20)$$

Is determined from equation (15) wind velocity component perpendicular to blade turbine generating similar situation model:

$$a_{\beta,s} = \sqrt{\frac{2\cdot g\cdot H_{m,s}}{\xi}} = \sqrt{\frac{2\cdot 9{,}81\cdot 1}{0{,}867}} = 4{,}758\, m/s \qquad (21)$$

Determine the area watered blade turbine model $S_{b,h,s}$:

$$Q_{v,s} = 1 m^3/s = a_{\beta,s}\cdot S_{b,h,s} \Rightarrow S_{b,h,s} = \frac{1}{a_{\beta,s}} = \frac{1}{4{,}758} = 0{,}21 m^2 \qquad (22)$$

Wetted area real turbine blade is:

$$S_{b,h} = m^2 \cdot S_{b,h,s} = 1329{,}6\cdot 0{,}21 = 279{,}45 m^2 \qquad (23)$$

The length of the blade wet air from the top to the hub is:

$$L_{b,h} = \frac{S_{b,h}}{\frac{\pi}{2}\cdot l_b} = \frac{279{,}45}{\frac{\pi}{2}\cdot 3{,}9} = 45{,}62\, m \qquad (24)$$

From equation (11) is recalculated blade perpendicular component of the wind speed:

$$\begin{aligned} a_\beta &= 2\cdot\left(\pi\cdot L_b\cdot l_b - S_{b,h}\right)\cdot\frac{n}{l_b} = \\ &= 2\cdot\left(\pi\cdot 49\cdot 3{,}9 - 279{,}45\right)\cdot\frac{0{,}0673}{3{,}9} = \\ &= 11{,}08\, m/s \end{aligned} \qquad (25)$$

Resume the calculation of the equation to equation (25) to converge. Finally we get: a_β = 10.86 m/s and $L_{b,h}$ = 46.62 m.

Wind turbine power corresponding to the wind speed is calculated similar to the relationship (14):

$$\begin{aligned} P_{12} &= 3\cdot\rho_{air}\cdot a_\beta^3\cdot\frac{\pi}{2}\cdot L_{b,h}\cdot l_b = \\ &= 3\cdot 1{,}225\cdot 10{,}86^3\cdot\frac{\pi}{2}\cdot 46{,}62\cdot 3{,}9 = \\ &= .345.500\, W = 1345{,}5\, kW \end{aligned} \qquad (26)$$

Similar powers are determined for increasing speeds up to maximum speed: P_{14} = 1465.7 kW, P_{16} = 1564.3 kW, P_{18} = 1646 kW, P_{20} = 1714.1 kW.

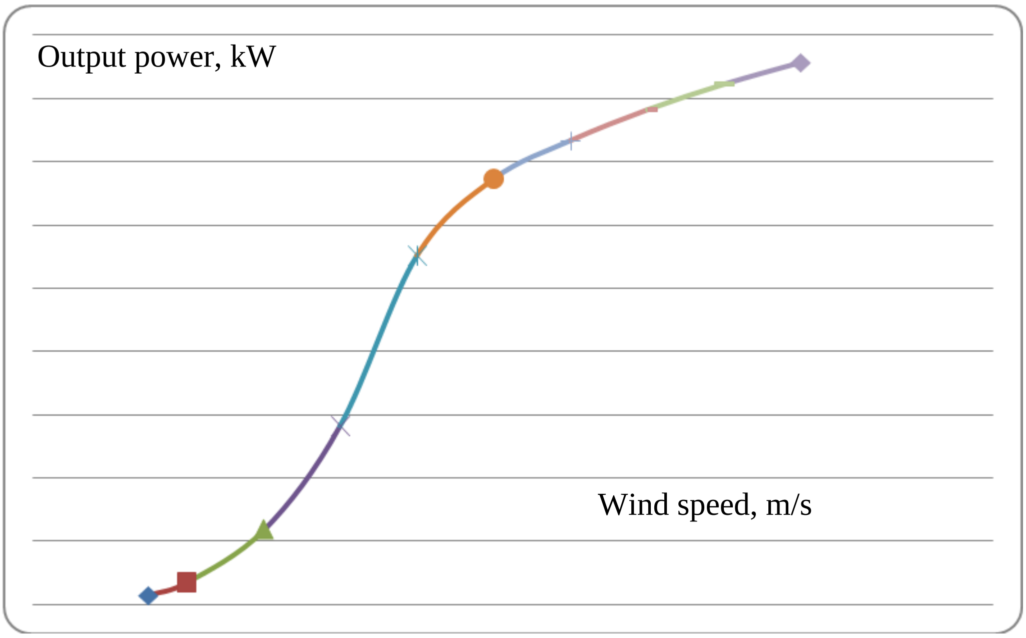

Figure 4. Output calculated power of the V100-1.8/2.0 MW wind turbine

Values deflection angle of rotation of the blade plan from the vertical position, taken at higher speeds than the reference wind speed are: $(90 - \beta_{12}) = 25$, $(90 - \beta_{14}) = 36$, $(90 - \beta_{16}) = 42$, $(90 - \beta_{18}) = 47$, $(90 - \beta_{20}) = 51$.

Graph power wind turbine, in kW, depending on the wind speed in m / s, obtained by the physical simulation model is (**Eroare! Fără sursă de referință.**).

4. Conclusions

The physical model gives results in the output power reported range. The obtained power evolution subject to the wind speed first follows the accelerating trend increase for low range wind velocities and then follows the decelerating trend increase for high range wind velocities. The physical model reports a less abrupt power increase trend than the reported profile. Angle values of hub deviation from vertical blade position are not reported in order to compare.

The physical model considers a simple geometry of the blade, as half cylinder, which is much less aerodynamically performing than if projected. However, results are not far from experimental curve reported by the producer company. The physical model computes independently, without adjustment, the turbine power, reporting only to the reference wind speed figured by the producer in his output power graphic representation.

The presented model may have a didactic impact in understanding and justifying the wind turbines functioning, and wish to be used as a confirmation of other reported results.

Acknowledgement

The authors would like to thank for the privilege of studying around regenerated resources in order to teach for their students in the Ecological University.

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